51.

$P(t) = \frac{300}{1+2^{4-t}}$

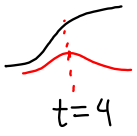
a) $t=0 \quad P(0) = \frac{300}{1+2^4} = \frac{300}{17} \approx 18$ students

b) $P'(t) = \frac{(1+2^{4-t})\cdot 0 - 300\,2^{4-t}\cdot \ln 2\,(-1)}{(1+2^{4-t})^2}$

$P'(4) = \frac{300 \cdot \ln 2}{(1+1)^2} \approx 52 \frac{\text{students}}{\text{day}}$

on day 4, 52 new students hear the rumor

c) when spread at max rate

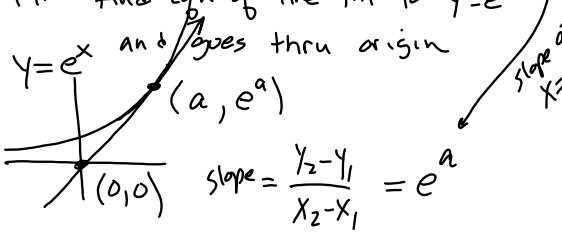



how fast 52 $\frac{\text{students}}{\text{day}}$

Sep 30-8:56 AM

49. find eqn of line tan to $y=e^x$ and goes thru origin

$y' = e^x$




$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = e^a$

$a=1 \quad \text{slope} = e^1 = e$

$y = e(x-0)+0$

$y = e\cdot x$

$\frac{e^a-0}{a-0} = e^a$

$\frac{e^a}{a} = e^a$

Sep 30-9:23 AM

logarithms

$y = \log_a x$ means $a^y = x$

$\log_{10} 100 = y$ means $10^y = 100$

$\log_{10} 100 = 2$ $\quad y = 2$

common log. $\log 100 = 2$

$\log 10 = 1 \qquad \log 1 = 0$

$\ln x = \log_e x \qquad \ln 1 = 0$

natural log.

Sep 30-9:38 AM

a method of logical arithmetic

Properties of logarithms

$\log(ab) = \log a + \log b$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

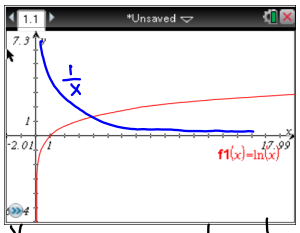
$\log a^n = n \log a$

Sep 30-9:43 AM

3.9b Derivatives of logarithms

Derivative of y=ln(x)

Sketch the graph of the derivative of y=ln(x)




$\frac{d}{dx}\ln x = \frac{1}{x}$

$y = \ln x$ means $e^y = x$

$y' = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$

$e^y \cdot y' = 1$

Oct 5-7:49 PM

Find dy/dx if

$y = \ln(2x)$ $\quad y' = \frac{1}{2x}\cdot 2 = \frac{1}{x}$

$y = \ln 2 + \ln x$ $\quad y' = 0 + \frac{1}{x}$

$y = \ln\frac{3}{x}$ $\quad y' = \frac{1}{\left(\frac{3}{x}\right)}\cdot\left(\frac{x\cdot 0 - 3\cdot 1}{x^2}\right)$

$y = \ln 3 - \ln x$

$y' = 0 - \frac{1}{x}$ $\quad = \frac{x}{3}\cdot\frac{-3}{x^2} = \frac{-1}{x}$

Oct 5-7:55 PM

Derivative of $y = \log_a x$ means $a^y = x$

$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$

$a^y \cdot \ln a \cdot y' = 1$

$y' = \frac{1}{a^y \ln a}$

chain rule version

$\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{dy}{dx}$

$y' = \frac{1}{x\ln a}$

Oct 5-7:57 PM

Find dy/dx if

$y = \log_2(\sin(x))$

$y' = \frac{1}{\sin x \ln 2}\cdot\cos x = \frac{\cos x}{\sin x\cdot\ln 2}$

$y = x^3\log_5(2x+1)$

$y' = x^3\cdot\frac{1}{(2x+1)\ln 5}\cdot 2 + 3x^2\cdot\log_5(2x+1)$

Oct 5-7:59 PM

Logarithmic Differentiation

Find dy/dx for $y = x^x$

take ln of both sides $\ln y = \ln x^x$

use properties of logarithms $\ln y = x \cdot \ln x$

Implicit diff. $\frac{1}{y} \cdot y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

solve for y $y' = (1 + \ln x) y$

Substitute for y $y' = (1 + \ln x) x^x$

Oct 5-8:03 PM

Find dy/dx

$$y = \frac{\sqrt{2x-1}(x+3)^5}{(x-7)^2}$$

Oct 7-8:56 AM